1 Administrative Topics

- I am returning quizzes today.
- Remember that this week’s project is not due until Friday night. Come to lab for extra help!
- We will be sending out grades for Project 4 on Wednesday.
- Stephanie has a conference in S.C. on Friday (assuming they have recovered from Hurricane Matthew), so Dale will be coming in to go over the HW and give the quiz.

2 Optimization

An important area of computer science (and mathematics!) is optimization. Given a function with one or more parameters, an optimization problem seeks to find the “best” value of the parameter(s).

In your project this week, the final task instructs you to “Use your simulation to figure out the percent of adult females that need to be dartered in order to hold the population steady over 200 years of simulation time. A steady population is defined as being more than 6500 and less than 8000 at the end of the simulation.”
This is an optimization problem - the darting percentage is a parameter for your simulation. Your simulation is outputting the “best” value if the population size is close to 7000.

We are going to talk about optimization today, motivated by the project, but not talking about the project specifically.

First, let me be a little more formal about how optimization problems are formulated. You need

1. A function to be optimized. This is called the \textit{objective function}

2. A description of the parameter space to be searched. (i.e. if you have a function that has one parameter \( x \), then give bounds on the values of \( x \)).

3. A definition of what it means to be “best.” There are three common definitions:

   - The best parameter value is the one that causes the objective function’s output to be the smallest (this is called a minimization problem)
   - The best parameter value is the one that causes the objective function’s output to be the largest (this is called a maximization problem)
   - The best parameter value is the one that causes the objective function’s output to be a particular value (or, since we are in the world of computers, “the closest to a particular value”)

\textbf{Continuous Optimization}

If the objective function is a continuous function, then we perform continuous optimization.

E.g. suppose we have a function

\[ f(x) = x^2 \]
and we want to find the value of $x$ that will give us an output of 2. (where is $x^2 == 4$?)

We are looking for a particular value. Typically, when we are looking for a particular value, we reformulate the problem so that we are looking for zero. I.e. we write our objective function as

$$c(x) = x^2 - 4$$

and look for values of $x$ that cause the output of the objective function $c(x)$ to output 0. We will look at values of $x$ between 0 and 100.

How could we go about finding the best value of $x$?

- Take advantage of our knowledge of math.
- Randomly Search: Test random values of $x$ and keep track of which $c(x)$ is closest to zero. You can refine your search as you go (search smaller areas once you have some “good” values)
- Linearly Search: Make a list of values to test for $x$ (start at the minimum value of $x$ allowed, stop at the maximum value of $x$ allowed, and use a step to linearly sample the points in between). Evaluate $c(x)$ at all these values and then use the value of $x$ that makes $c(x)$ closest to zero.

**Discrete Optimization**

Some of the same techniques that we used above can be used when the values of $x$ are restricted to discrete values (e.g. integers). If the objective function is a discrete function, then we perform discrete optimization. e.g. we have a function whose inputs can be integers between 0 and 9 and whose output is in the range 0.0 to 3.0). We can represent such a function with code to determine the function (it could be math) or with just at table.

- Take advantage of our knowledge of math.
• Randomly Search: Test random values of $x$ and keep track of which $c(x)$ is closest to zero.

• Linearly Search: Make a list of values to test for $x$ and use the value of $x$ that makes $c(x)$ closest to zero. This could mean checking every potential value.