Analysis of Algorithms
CS 375, Fall 2018
Homework 14
Due **AT THE BEGINNING OF CLASS** Monday, November 12

- **Reading Assignment:** From your textbook (Levitin), Please finish reading Chapter 5.
- *A general note:* When writing up your homework, please write neatly and explain your answers clearly, giving all details needed to make your answers easy to understand. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

Exercises

1. Exercise 5.2.11. **HINT:** This exercise is in the Quicksort chapter for a reason, and that reason is Partition! Think about how to use the idea of partitioning elements as part of this algorithm.

2. By popular request, here are a few exercises on properties of logarithms! You'll be asked to show that a few properties of logarithms are true. The objective is to help you become more familiar with some properties of logarithms that are especially important for analysis of recursive algorithms.

   As an example, here's how we might show that $\log_b xy = \log_b x + \log_b y$. In the below, recall from the definition of logarithm that a logarithm really stands for an exponent—$x = log_b n$ means that $b^x = n$, i.e., x is the exponent we raise b to, to get n.

   > To make it easier to talk about raising some relevant expressions to a power, we'll introduce a few variables: let $k = \log_b xy$, $\ell = \log_b x$, and $m = \log_b y$. Then, by the definition of logarithm (above), $b^k = xy$, $b^\ell = x$, and $b^m = y$. (Note: Do you see why?) Therefore, $b^k = b^\ell b^m = b^{\ell+m}$, and thus, by standard properties of exponents, $k = \ell + m$, which is what we had set out to show.

   Now, pick 2 of the following 3 properties of logarithms (your choice!) and show that they are true. (If you turn in all 3 of them, only your best 2 will be counted for your grade, so I hope you'll give all 3 of them a try!)

   Please give detailed explanations, as shown in the example above! Assume that a, b, c, n are positive real numbers (but note—they are not necessarily integers).

   (a) $\log_b a^n = n \log_b a$.

   (b) $\log_b n = \frac{\log_c n}{\log_c b}$. (This shows that changing the base of a logarithm—say, from $log_b n$ to $log_c n$—really means multiplying by a constant factor, because this could be rewritten as $log_b n = \frac{1}{log_c b} \cdot log_c n$.)

   (c) $a^{\log_b n} = n^{\log_b a}$. (This result is sometimes called the "log-switching theorem" since it says that we can "switch" a and n.)