Exercises

1. Apply Kruskal’s algorithm to the following graph, to find a minimum spanning tree. Show your work: To show that you understand Kruskal’s algorithm, show the order in which each edge is added and, for each added edge, give a very brief (1 sentence or less is fine!) explanation of the reasons behind choosing that edge.

2. Bridge Crossing Revisited! Consider the generalization of the bridge crossing exercise earlier in the semester (HW1, exercise 6) in which there are \( n > 1 \) people whose bridge crossing times are \( t_1, t_2, \ldots, t_n \). All the other conditions of the problem are the same as before: at most two people at a time can cross the bridge (and they move with the speed of the slower of the two) and they must carry with them the only flashlight the group has.

   (a) Design a greedy algorithm for getting the entire group across the bridge, with the goal of minimizing the total time for the group to cross. Give a very brief explanation of what makes it a greedy algorithm.
(b) Show that the greedy algorithm does not always yield an optimal solution (i.e., a minimal crossing time) for every instance of this problem by giving a concrete counterexample with the smallest number of people for which it is not optimal.

3. Exercise 34-1 in CLRS (pages 1101–1102) defines the independent-set problem. As presented, it is an optimization problem. Formulate a related independent-set decision problem by giving the input / output specifications of the decision variant, and explain briefly (1 sentence or so should suffice) how it relates to the optimization problem.

4. The bin-packing problem is an optimization problem specified as follows:

   **Input** Set \( S = \{i_1, \ldots, i_n\} \) of \( n \) items, where item \( i_z \) has associated rational-number size \( s_z \) \((0 < s_z \leq 1)\)

   **Output** Integer \( m \), which is the smallest integer such that all items in \( S \) can fit into \( m \) bins of size 1.

   For example, if \( S \) has three items all with size \( \frac{1}{2} \), they could fit in 2 bins of size 1—two items in one bin, one item in the third. If \( S \) has five items of sizes \( \frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4} \), they could fit in three bins of size 1, but they could not fit in two bins of size 1.

   Formulate a related bin-packing decision problem by giving the input / output specifications of the decision variant, and explain briefly (1 sentence or so should suffice) how it relates to the optimization problem.