Theory of Computation
CS 378
Fall 2019
Problem Set 7
Due BY THE END OF THE DAY Friday, December 6

• A general note: When writing up your homework, please explain your arguments clearly and write neatly. Graders may not award credit to incomplete or illegible solutions. Clear communication is the point, on every assignment.

1. $L_{\text{union}}$

Consider the following language:

$$L_{\text{union}} = \{ < M_1, M_2, M_3 > | M_1, M_2, M_3 \text{ are TMs and } L(M_1) \cup L(M_2) = L(M_3) \}$$

Prove by reduction that this language is not recursive.

2. Oracles! You have been hired as the chief architect at International Turing Machines Corporation (ITM), a major manufacturer of theoretical computing machines. One day you receive a package in the mail postmarked from Delphi, Greece. The package contains a black box labeled “An Oracle for the Halting Problem.” The instructions explain that the oracle may be “connected” to any Turing Machine as follows. You connect the oracle to your Turing Machine by simply introducing three new states to your TM: $q_{\text{oracle}}, q_{\text{Yes}},$ and $q_{\text{No}}$. When your Turing machine enters state $q_{\text{oracle}}$ with a machine-word pair $< M, w >$ at the front of its input tape, the oracle will instantly cause your TM to enter state $q_{\text{Yes}}$ if Turing machine $M$ halts on $w$, and will cause your machine to enter state $q_{\text{No}}$ otherwise.

You are very excited by your good fortune. Show that even the diagonal language, $L_d$, is decidable with this oracle.