Analysis of Algorithms
CS 375, Spring 2019
Homework 3
Due **AT THE BEGINNING OF CLASS** Wednesday, February 20

- From your textbook (CLRS), please be sure to read Chapter 2.

- Unless otherwise specified, exercises will be from the CLRS textbook and will be named on HW assignments by exercise number used in the book.

- Standard disclaimer: All back-stories about the fictional "Portland Institute of Technology" and its faculty members are completely gratuitous!

- *A general note:* When writing up your homework, please write neatly and **explain your answers clearly**, giving all details needed to make your answers easy to understand. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

Exercises

1. For each of the following pairs of functions, indicate whether the order of growth of the first function of the pairs is lower, the same, or higher (to within a constant multiple) than the order of growth of the second function. As always, give a short explanation (a sentence or two should suffice) of how you came up with your answer.

   (a) $n(n+1)$ and $2000n^2$

   (b) $100n^2$ and $0.01n^3$

   (c) $\lg n$ and $\ln n$

   (d) 2^{n-1} and 2^n

   (e) $(n-1)!$ and $n!$

2. For each of the following assertions, say if they are True or False and give a short explanation of your answer.

   (a) $\frac{n(n+1)}{2} \in O(n^3)$

   (b) $\frac{n(n+1)}{2} \in O(n^2)$

   (c) $\frac{n(n+1)}{2} \in \Theta(n^3)$

   (d) $\frac{n(n+1)}{2} \in \Omega(n)$

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3. Prof. Sue Persmart in the CS Department at Portland Institute of Technology (motto: "Our CS Department is the PIT's!") likes to tell a story about the invention of chess.

   (a) According to legend, the game of chess we invented long ago in India by a certain sage. When he took the invention to his king, the king liked the game so much that he offered the inventor any reward he wanted. The inventor asked for some grain to be obtained as follows: Just one grain of wheat was to be placed on the first square of the chessboard, then two grains on the second square, four grains on the third square, eight grains on the fourth, etc., until all 64 squares had been filled.

   If it took 1 second to count each grain of wheat, how long would it take to count all the grains of wheat due to the sage?

   (b) What if, instead of doubling the number of grains for each square of the chessboard, the inventor asked for adding two grains. Then (assuming again that it took 1 second to count each grain) how long would it take to count all the grains of wheat due to the sage?