Analysis of Algorithms
CS 375, Spring 2019
Homework 6
Due **AT THE BEGINNING OF CLASS** Wednesday, March 6

- From your textbook (CLRS), please read Chapter 4, pages 65–67 and 88–92.
- *A general note:* When writing up your homework, please write neatly and **explain your answers clearly**, giving all details needed to make your answers easy to understand. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

Exercises

1. Solve the following recurrence relations and give the Θ class of the solution.
   (a) $T(n) = T(n-1) + 5$ for $n > 1$; $T(1) = 0$.
   (b) $T(n) = 3T(n-1)$ for $n > 1$; $T(1) = 4$.
   (c) $T(n) = T(n-1) + n$ for $n > 0$; $T(0) = 0$.
   (d) $T(n) = T(n/2) + n$ for $n > 1$; $T(1) = 1$. (Assume n is of the form 2^k for this exercise.)
   (e) $T(n) = T(n/3) + 1$ for $n > 1$; $T(1) = 1$. (Assume n is of the form 3^k for this exercise.)

2. An algorithm to compute the sum of the cubes of the first n positive integers—i.e., the sum $1^3 + 2^3 + \cdots + n^3$—would be defined by the following specifications:

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// Input: n, a positive integer
// Output: The sum of the cubes of the first n positive integers
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   (a) Write a recursive algorithm to compute the sum of the cubes of the first n positive integers as specified above. (You may not use global variables, extra parameters, default parameters, helper functions, etc.—your work must be a single function that meets the above specifications.) To compute a value x^3, be sure to explicitly compute $x \cdot x \cdot x$—i.e., multiply the value by itself. (This restriction is important for the next part of the exercise.)
   (b) The multiplication operation could be a reasonable barometer operation for this algorithm. Set up and solve a recurrence for the number of multiplications performed by the algorithm, in terms of input n.