

Assignment 10

3SAT

Input: Boolean Variables $x_1, \dots, x_n$ and clauses $C_1, \dots, C_m$ such that each clause is 3 literals (x_i or $\bar{x}_i$) ORed together.

Solution: TRUE if there is a setting of the variables so that $C_1 \wedge C_2 \wedge \dots \wedge C_m$ evaluates to TRUE, and FALSE otherwise.

VERTEX COVER (VC)

Input: A graph G and integer k

Solution: TRUE if there are k vertices in G that touch every edge, and FALSE otherwise.

1. Consider the input I_1 given by

$$(x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_1 \vee x_2 \vee \bar{x}_3) \\ \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)$$

Find a satisfying assignment for this formula.

Solution: $x_1 = T$, $x_2 = F$, and $x_3 = T$ should work.

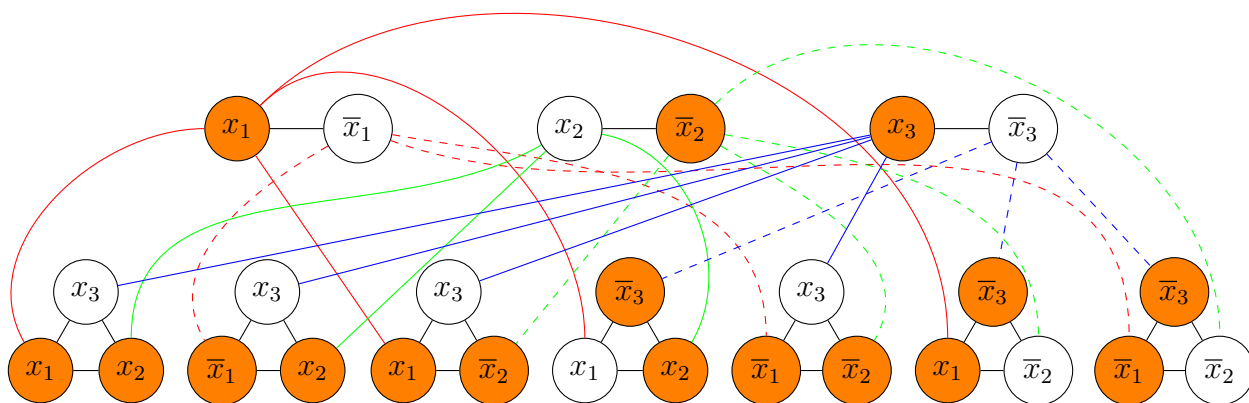
2. Convince yourself that there is no satisfying assignment for the input

$$I_2 = I_1 \wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_3)$$

Solution: There's $2^3 = 8$ possible assignments - for each one, there is a clause that negates all 3 true literals.

3. In class we showed that $3SAT \leq_p VC$: compute the reduction on the input I_1 and find a vertex cover of size $k = 3 + 2 * 7 = 17$

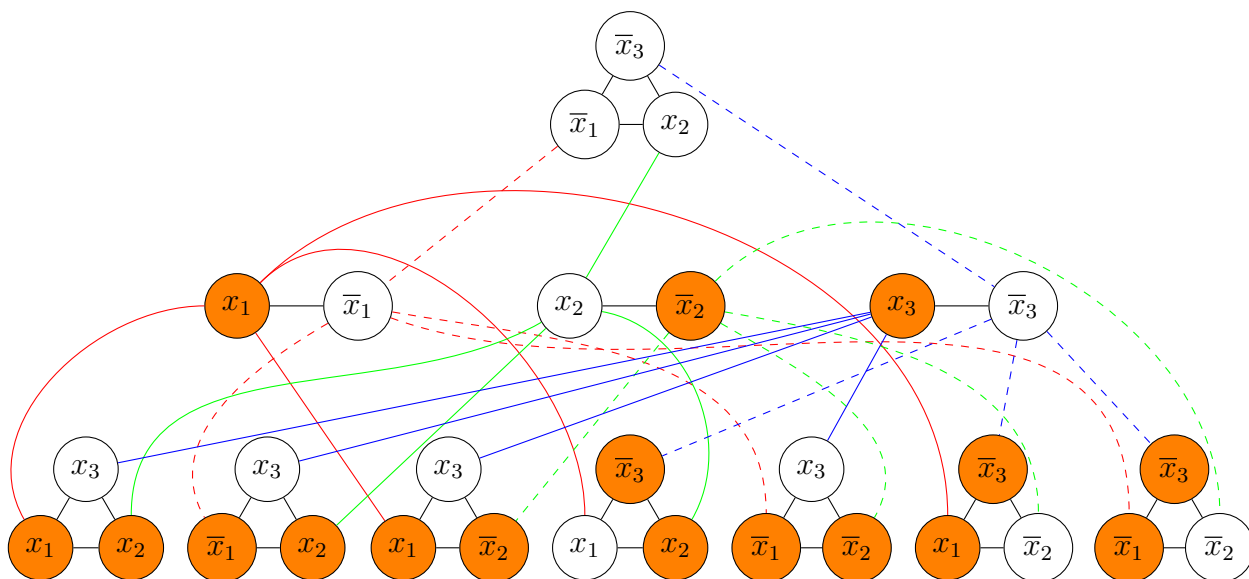
Solution:



4. Compute the reduction for I_2 (this just amounts to adding 3 more vertices and 6 more edges) and convince yourself that there is no vertex cover of size $k = 19$.

Solution:

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There is no way to pick 2 more vertices from the top triangle to cover the 6 new edges. While this doesn't technically rule out all other possibilities, going over the argument from class will rule out there being a solution to this (as it would imply a solution to the equivalent 3SAT instance, which we know has no solution by exhaustion of possibilities).