

Assignment 3 Solutions

1. This problem concerns the Minimum Spanning Tree problem.

- (a) Consider a weighted graph G where there is one edge e that has maximal weight (so no edge in the graph has weight greater than or equal to the weight of e) and belongs to a cyclic subgraph of G (so there is some cycle of edges that exists in G where e is a part of that cycle). Convince me that no MST in G can include e .

Answer: Suppose T is a spanning tree that includes e , then there is some other edge e' in the cycle that it must not have included. By swapping e and e' , we get a new spanning tree that must have lower weight, since e was the heaviest edge in the cycle. Hence, any spanning tree that contains e cannot be of minimal weight.

- (b) Consider a weighted graph where there is one edge e that has minimal weight (so no edge in the graph has weight less than or equal to the weight of e). Convince me that every MST in G must include e .

Answer: Suppose T is a spanning tree that doesn't include e . By adding e to T , we must get some cycle - let e' be the heaviest edge in this cycle. Consider the tree formed by deleting e' from T and adding e : this operation guarantees that we have a spanning tree, whose weight must be less than the weight of T since the weight of e must be less than the weight of e' . Thus any spanning tree that doesn't include e would lose weight by including e and deleting some other edge; thus any minimal weight spanning tree must include e .