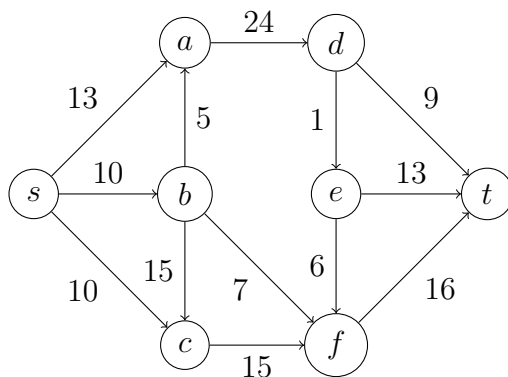


Assignment 9 Solutions

Recall the statement of the Ford-Fulkerson algorithm as follows:

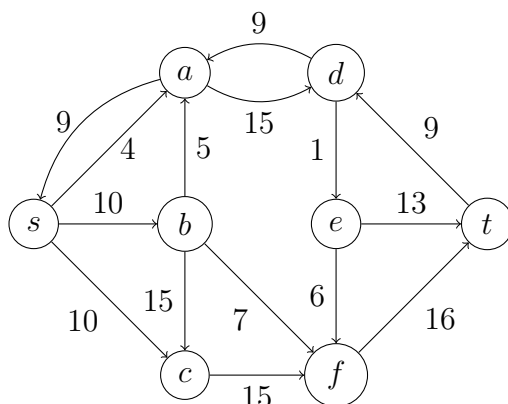
```
def residualGraph(graph  $G$ , flow  $f$ ):  
    Initialize a new graph  $G_f$  with the same vertices as  $G$   
    for each edge  $e$  in  $G$ :  
        Let  $e^{G_f}$  denote the corresponding edge in  $G_f$   
        Set  $c_{e^{G_f}} = c_e - f(e)$   
        If  $f(e) > 0$ :  
            Let  $e^R$  denote the reverse edge of  $e^{G_f}$ .  
            Set  $c_{e^R} = f(e)$ .  
    return  $G_f$   
  
def FordFulkerson( $G$ ,  $s$ ,  $t$ ):  
    Initialize  $f$  as zero flow.  
    while there is an  $s \rightarrow t$  path  $P$  in  $G_f$  where the minimum  
        capacity  $c$  over all the edges in  $P$  is bigger  
        than 0:  
        Set  $f(e) += c$  for all edges  $e$  in  $P$ .
```

Trace through the FordFulkerson algorithm on the following input:



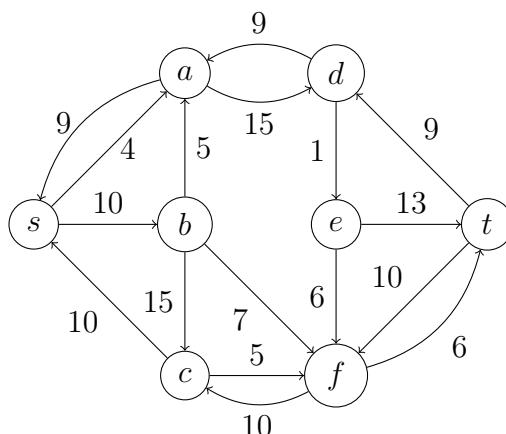
After determining the maximum flow, find an s - t cut with equal cut value.

Solution: First $s \rightarrow t$ path: $s \rightarrow a \rightarrow d \rightarrow t$. The bottleneck capacity is 9, so we push that much, giving us a residual graph of

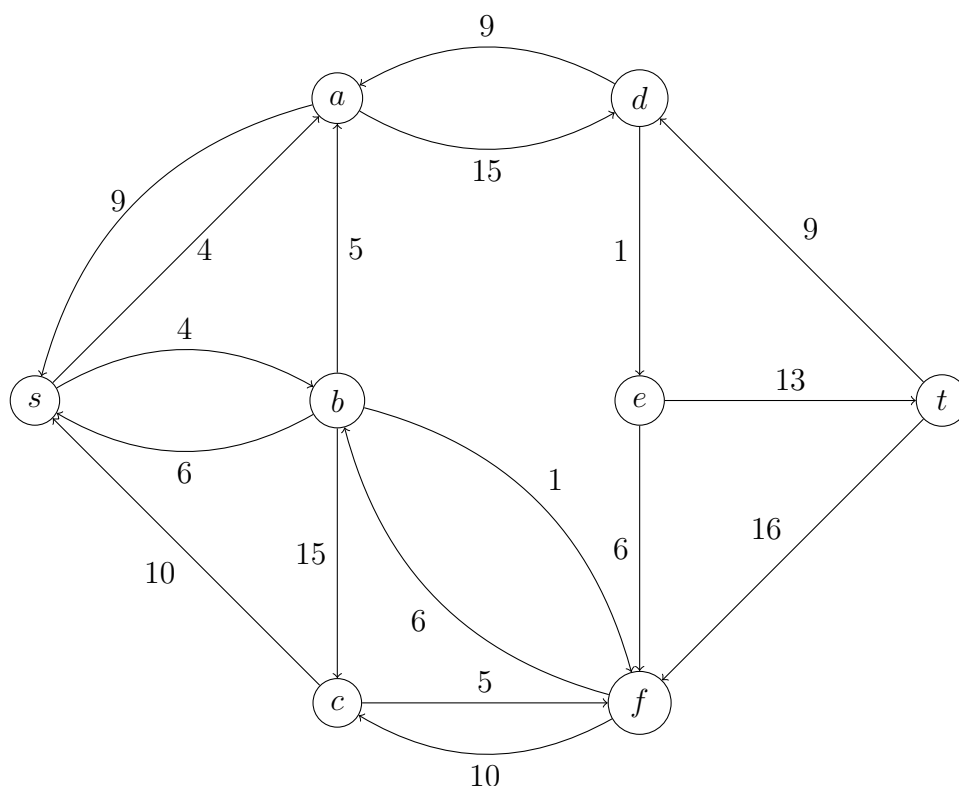


Assignment 9 Solutions

For our next $s \rightarrow t$ path in the residual graph, we use the path $s \rightarrow c \rightarrow f \rightarrow t$, giving us a bottleneck of 10 capacity. By pushing 10 flow along this path, we get a residual graph of:

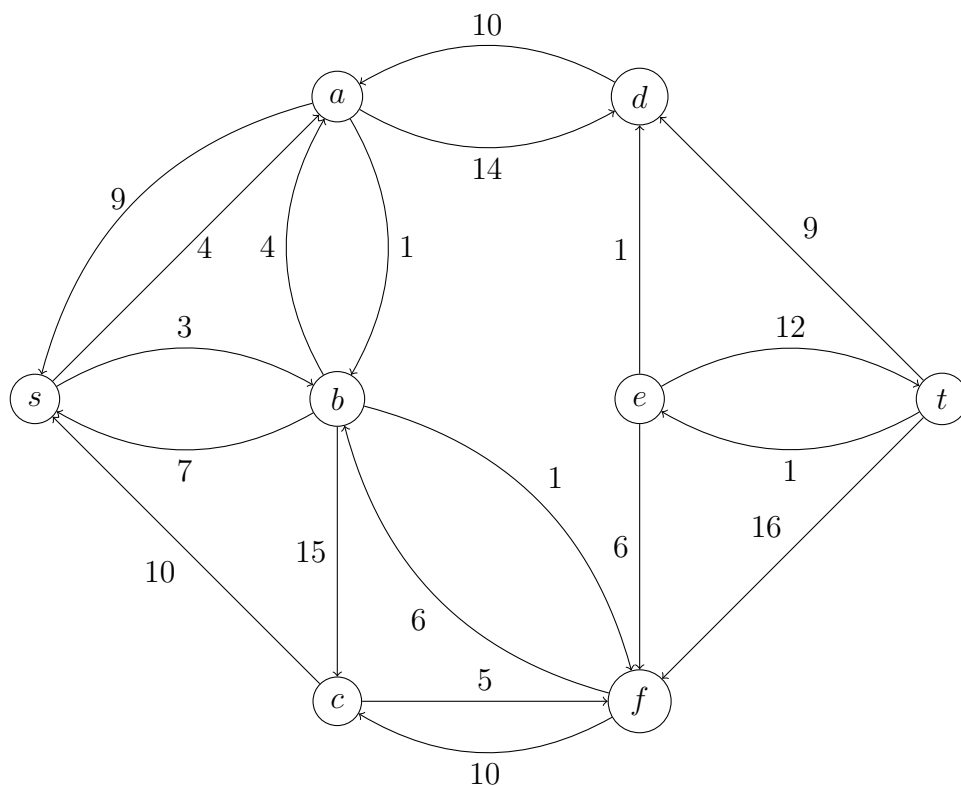


For our next $s \rightarrow t$ path in the residual graph, we use the path $s \rightarrow b \rightarrow f \rightarrow t$, with a bottleneck capacity of 6. By pushing 6 flow along this path we get the following residual graph:



For our final $s \rightarrow t$ path, we use $s \rightarrow b \rightarrow a \rightarrow d \rightarrow e \rightarrow t$, where the bottleneck capacity is 1. By pushing 1 unit of flow on that path, we get the residual graph

Assignment 9 Solutions



The resultant value of this flow is $9 + 10 + 6 + 1 = 26$.

Consider the $s - t$ cut given by $A^* = \{s, a, b, c, d, f\}$ and $B^* = \{e, t\}$. Then the cut value of (A^*, B^*) is $9 + 1 + 16 = 26$. Hence we have a flow value equal to a cut value, meaning that the flow value is maximal and the cut value is minimal.