

Theory of Computation
CS 378
Fall 2022

Problem Set 5

Due by **11:59pm** on **Friday, December 2**

- Standard file naming and HW submission conventions apply: Please submit your type-written answers in a PDF file named

CS378_PS5_<userid>.pdf

where <userid> is replaced by your Colby userid. Please use your full userid, including your class year. Place the properly named file in your **Private** directory in your CS378 filespace on **filer**.

- Please read Sipser Ch. 3 and 4.
 - As always, answers to all exercises should be accompanied by explanations; brief explanations sometimes suffice, but all answers should be explained. Answers without explanations may not be awarded full credit.
 - *A general note:* Neatly written (or typeset) solutions, with enough blank space on the page to allow graders to write comments before returning the papers, are greatly appreciated!
1. **Space Warp Turing Machines!** NASA would like to send a Turing machine on a mission to a distant galaxy where there are holes in the space continuum. A *space warp* TM is a TM with a two-way infinite tape that operates under the following adverse conditions. At any moment during its computation, it may accidentally fall into a “space warp”, at which point the tape head is immediately transported to an arbitrary location on the two-way infinite tape. At the same time, its state is changed (involuntarily!) to a special state q_{warp} . Fortunately, NASA has determined that the number of space warps occurring during any computation is some fixed constant α , although no one has yet determined the value of α . As NASA’s Director of Theoretical Computer Science, your job is to show that any computation carried out by a regular TM can be carried out by a space warp TM. In other words, give a careful and complete (English) description of how a space warp TM can be programmed to recover from any interruption and continue its computation. Note that the machine may be interrupted while it is trying to recover from a previous interruption!
 2. In each case below, determine whether the given set is countable or uncountable. As always, prove your answer. ($\mathbb{N}$ refers to the set of natural numbers.)
 - (a) The set of all finite subsets of $\mathbb{N}$. In addition, explain why the diagonal argument cannot be used to show that this set is uncountable.
 - (b) The set of all functions from $\mathbb{N}$ to $\{0, 1\}$.
 - (c) The set of all functions from $\{0, 1\}$ to $\mathbb{N}$.
 - (d) The set of all regular languages over $\{0, 1\}$.