

Theory of Computation
CS 378
Fall 2022

Problem Set 6

Due by 11:59pm on Friday, December 9

- Standard file naming and HW submission conventions apply: Please submit your type-written answers in a PDF file named

CS378_PS6_<userid>.pdf

where <userid> is replaced by your Colby userid. Please use your full userid, including your class year. Place the properly named file in your **Private** directory in your CS378 filespace on **filer**.

- Please read Sipser Ch. 4, 5.1, and 5.3.
- **NOTE:** It may be helpful to wait until our class meeting on Tuesday, Dec. 6, to do the reduction for Ex. 1 below.
- As always, answers to all exercises should be accompanied by explanations; brief explanations sometimes suffice, but all answers should be explained. Answers without explanations may not be awarded full credit.
- *A general note:* Neatly written (or typeset) solutions, with enough blank space on the page to allow graders to write comments before returning the papers, are greatly appreciated!

1. L_{union}

Consider the following language:

$$L_{\text{union}} = \{ \langle M_1, M_2, M_3 \rangle \mid M_1, M_2, M_3 \text{ are TMs and } \mathcal{L}(M_1) \cup \mathcal{L}(M_2) = \mathcal{L}(M_3) \}$$

Prove by *reduction* that this language is not recursive. You may reduce from any language shown in class to be undecidable.

2. **Oracles!** You have been hired as the chief architect at International Turing Machines Corporation (ITM), a major manufacturer of theoretical computing machines. One day you receive a package in the mail postmarked from Delphi, Greece. The package contains a black box labeled “An Oracle for the Halting Problem.” The instructions explain that the oracle may be “connected” to any Turing Machine as follows. You connect the oracle to your Turing Machine by simply introducing three new states to your TM: q_{oracle} , q_{Yes} , and q_{No} . When your Turing machine enters state q_{oracle} with a machine-word pair $\langle M, w \rangle$ at the front of its input tape, the oracle will instantly cause your TM to enter state q_{Yes} if Turing machine M halts on w , and will cause your machine to enter state q_{No} otherwise.

You are **very excited** by your good fortune. Show that even the diagonal language, L_d , is decidable with this oracle.