

CS375 Small Assignment 11

Due Nov 20th 10PM ET

Recall the MAXFLOW problem from class:

MAXFLOW

Input: A graph $G = (V, E)$ with vertices V and directed edges E with capacities $c(u, v) \geq 0$ for each edge (u, v) in E and two marked vertices s and t (we'll assume there are no edges into s or out of t).

Output: The maximum flow on the graph.

Constraints:

1. For each vertex v other than s and t it holds that $\sum_u f(u, v) = \sum_w f(v, w)$:
ie, the flow into all vertices equals the flow out.
2. For each edge (u, v) in E it holds that $f(u, v) \leq c(u, v)$.

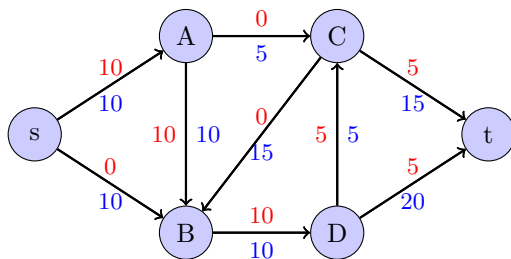
In order to define the Ford-Fulkerson algorithm in class, we defined the residual graph of G with respect to f as $G_f = (V, E_f)$ where $c_f(u, v) = c(u, v) - f(u, v)$, $c_f(v, u) = f(u, v)$, and E_f is all the edges with positive residual capacity c_f (note that this may remove some edges from E and may include some reverse edges).

We defined the Ford-Fulkerson algorithm as follows:

Ford-Fulkerson($G = (V, E), s, t$):

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initialize  $f(u, v) = 0$  for all  $(u, v)$  in  $E$ 
while there exists a path  $P$  from  $s$  to  $t$  in  $G_f$ :
    let  $(u, v)$  be the edge of minimal residual capacity in  $P$ 
    Increase the flow  $f$  by  $c_f(u, v)$  for every edge in  $P$ .
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Consider the following graph with capacities marked in blue, on which I've already pushed some flow (in red):



1. Using this as a starting point (so don't reset the flow to 0), enter the while loop in the FORD-FULKERSON algorithm, trace through what happens, and determine the max flow.
2. Identify an S-T cut with cut-capacity equal to the maximal flow.
3. (This question has nothing to do with the above two) Consider the following problem:

DISJOINTPATHS

Input: A directed graph G and two marked vertices s and t

Output: The maximum number of edge-disjoint paths from s to t

Constraints: Here, two paths are called edge-disjoint if they have no over-lapping edges (though they are allowed to have over-lapping vertices)

Show that $\text{DISJOINTPATHS} \leq_p \text{MAXFLOW}$.