

CS375 Weekly Assignment 1

Due Sep 16th 10PM ET

A general note for CS375: When writing up your homework, please write neatly and explain your answers clearly, giving all details needed to make your answers easy to understand. Graders may not award full credit to incomplete or illegible solutions.

You are encouraged to discuss the problems with your classmates, **but your submission should be your own**. When writing your solutions up, you should not be looking at any other student's submission.

Clear communication **is the point**, on every assignment. In general in CS375, unless explicitly specified otherwise, answers should be accompanied by explanations. Answers without explanations may not receive full credit. Please feel free to ask me any questions about explanations that might come up!

Necessary acknowledgements: many of these questions are reproductions or adaptations of questions of Eric Aaron's devising. Others are from our course texts as listed on the syllabus.

1. Pick 2 of the following and prove them using induction.

i) $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

ii) $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$

iii) $\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r}$ ($\leftarrow$ we're going to use this one in class later)

iv) $\sum_{i=1}^n (2i-1) = n^2$

v) $\sum_{i=1}^n 2i = n(n+1)$

vi) $\sum_{i=1}^n F_i = F_{n+2} - 1$ where F_i is the i^{th} Fibonacci number ($F_1 = F_2 = 1$, $F_{i>2} = F_{i-1} + F_{i-2}$)

vii) $n! > 2^n$ for $n \geq 4$ ($\leftarrow$ useful for problem 3)

2. Pick 15 of the following functions and list them in order of asymptotic growth rate from least to greatest. Eg if the only functions were n^3 , 1 , n^2 , n , then the correct order would be 1 , n , n^2 , n^3 , as $1 = O(n)$, $n = O(n^2)$, and $n^2 = O(n^3)$. For any two that you pick that are Θ of each other, point them out. Note that $\lg$ is shorthand for $\log_2$. Of the 15 that you pick, you must pick: n^2 , $n!$, n^3 , $\lg^2 n$, $\lg n$, $2^{\lg n}$, $4^{\lg n}$, $(n+1)!$, $\sqrt{n}$, n , 2^n , $n \lg n$ (which means you should pick 3 more on top of these).

$$\begin{array}{cccccc}
 \lg(\lg^* n) & 2^{\lg^* n} & (\sqrt{2})^{\lg n} & n^2 & n! & (\lg n)! \\
 \left(\frac{3}{2}\right)^n & n^3 & \lg^2 n & \lg(n!) & 2^{2^n} & n^{\frac{1}{\lg n}} \\
 \lg \lg n & \lg^* n & n2^n & n^{\lg \lg n} & \lg n & 1 \\
 2^{\lg n} & (\lg n)^{\lg n} & e^n & 4^{\lg n} & (n+1)! & \sqrt{n} \\
 \lg^*(\lg n) & 2^{\sqrt{2 \lg n}} & n & 2^n & n \lg n & 2^{2^{n+1}}
 \end{array}$$

We almost certainly won't use $\lg^*$ this semester, and I can't imagine many (if any) of you have seen it before, but $\lg^*(n)$ = the number of times needed to compose $\lg$ to get to a value less than 1. Eg: $\lg^* 2 = 1$, $\lg^* 4 = 2$, $\lg^* 16 = 3$, $\lg^* 65536 = 4$, $\lg^*(2^{65536}) = 5$. It is an EXTREMELY slowly growing function.

You should show your work for each sequential function you write.