David Soltero CS375 Oct 19, 2026

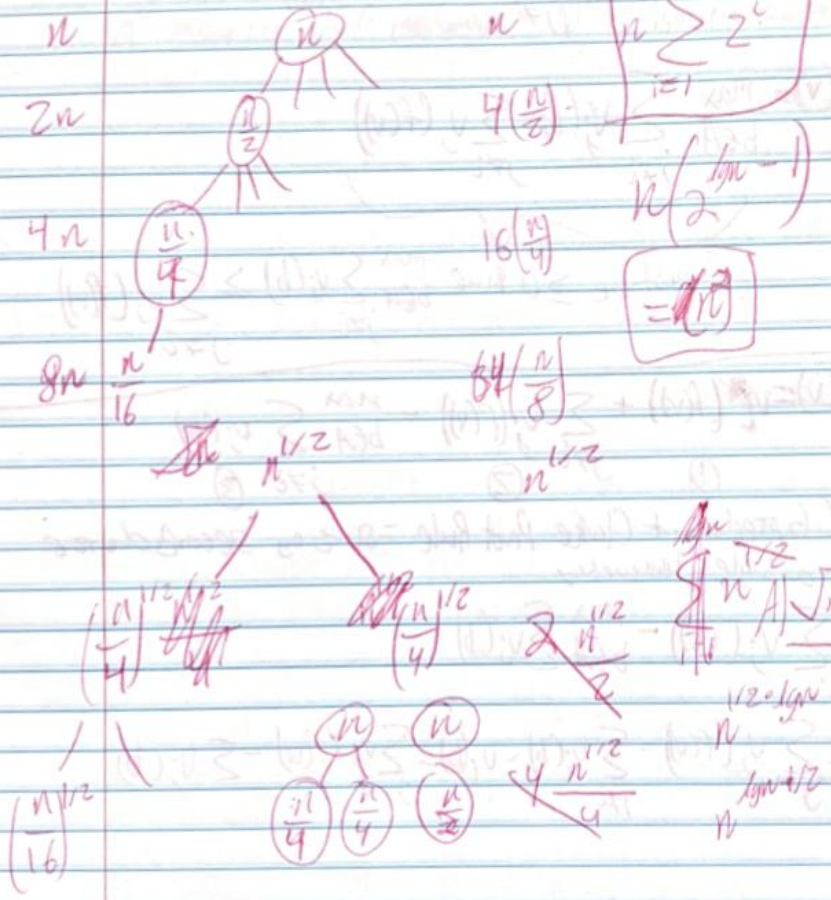
Today:

$A(n) = 2A(\frac{n}{4}) + \sqrt{n}$ | $B(n) = 2B(\frac{n}{4}) + n$ | $C(n) = 2C(\frac{n}{4}) + n^2$

$D(n) = 3D(\frac{n}{3}) + \sqrt{n}$ | $E(n) = 3E(\frac{n}{3}) + n$ | $F(n) = 3F(\frac{n}{3}) + n^2$

$G(n) = 4G(\frac{n}{2}) + \sqrt{n}$ | $H(n) = 4H(\frac{n}{2}) + n$ | $I(n) = 4I(\frac{n}{2}) + n^2$

$H(n) = 4(H(\frac{n}{2})) + n$



Master Theorem: If $T(n) = aT(\frac{n}{b}) + f(n)$, let $c^* = \log_b a$

If $f(n) = O(n^c)$ for some $c < c^*$, then $T(n) = \Theta(n^{c^*})$

If $f(n) = \Theta(n^{c^*}(\log n)^k)$ for some $k \geq 0$, then $T(n) = \Theta(n^{c^*}\log^{k+1} n)$

constant work per level $= \Theta(f(n)\log n)$

If $f(n) = \Omega(n^c)$ for some $c > c^*$ and if $af(\frac{n}{b}) < kf(n)$ for some $k > 0$, then $T(n) = \Theta(f(n))$.

Strong vs weak induction

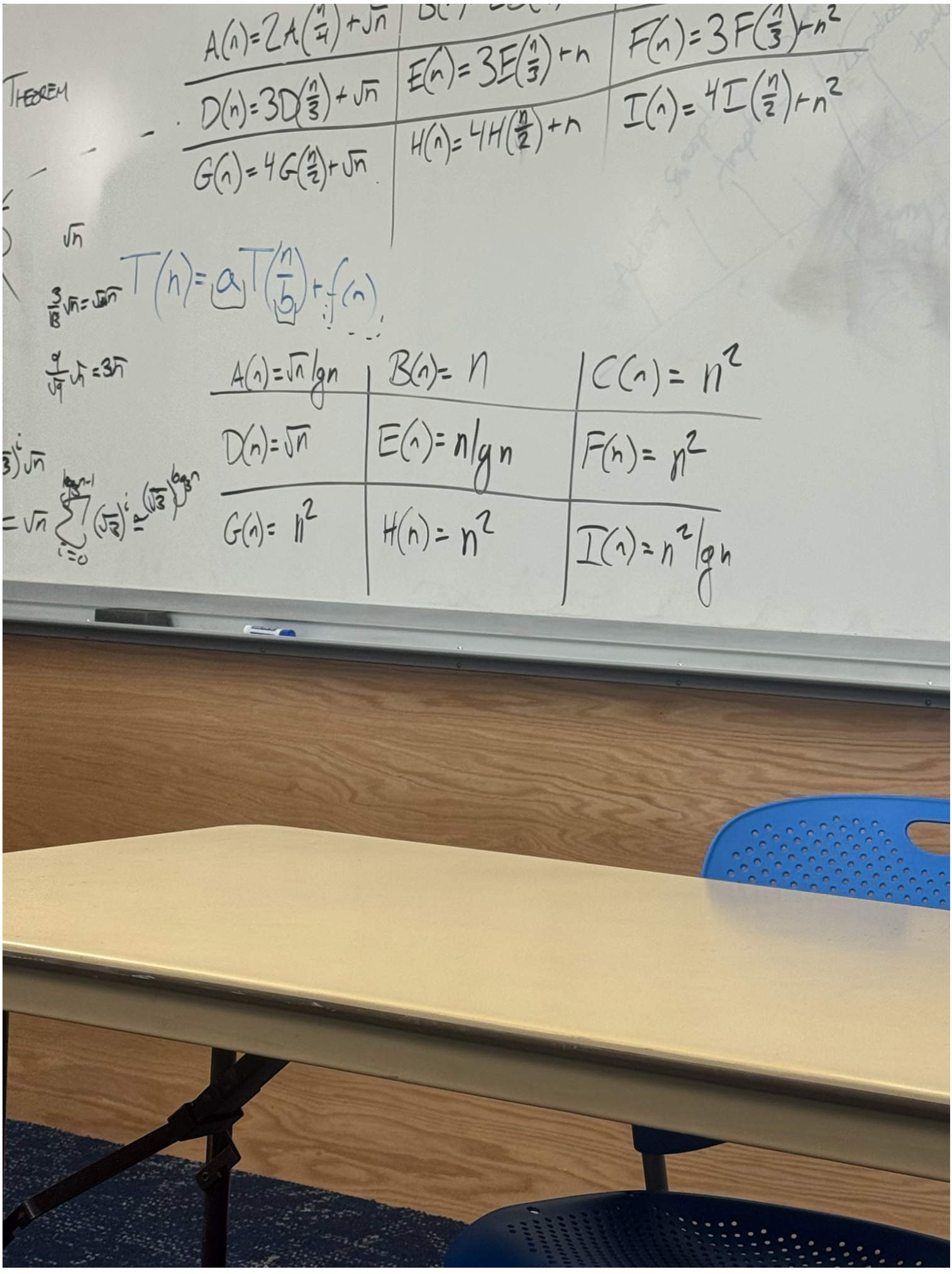
Weak Induction: If I know for that some formula $f(n)$ that

(1). $f(c)$ holds for some constant c } prove these two for
(2). If $f(k)$ holds then $f(k+1)$ holds, } all natural numbers

then $f(n)$ holds for all $n \geq c$

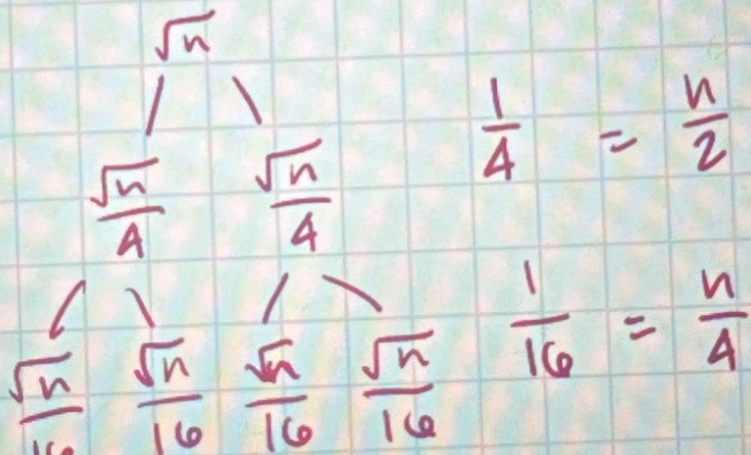
Strong Induction: Instead of (2), then is if $f(i)$ holds for all $c \leq i \leq k$, then $f(k+1)$ holds, then $f(n)$ holds for all $n \geq c$.

$A(n) = 2A\left(\frac{n}{4}\right) + \sqrt{n}$	$B(n) = 2B\left(\frac{n}{4}\right) + n$	$C(n) = 2C\left(\frac{1}{4}\right) + n^2$
$D(n) = 3D\left(\frac{n}{3}\right) + \sqrt{n}$	$E(n) = 3E\left(\frac{n}{3}\right) + n$	$F(n) = 3F\left(\frac{1}{3}\right) + n^2$
$G(n) = 4D\left(\frac{n}{2}\right) + \sqrt{n}$	$H(n) = 4H\left(\frac{n}{2}\right) + n$	$I(n) = 4I\left(\frac{n}{2}\right) + n^2$


THEOREM
$A(n) = 2A\left(\frac{n}{4}\right) + \sqrt{n}$
$D(n) = 3D\left(\frac{n}{3}\right) + \sqrt{n}$
$G(n) = 4G\left(\frac{n}{2}\right) + \sqrt{n}$
$E(n) = 3E\left(\frac{n}{3}\right) + n$
$H(n) = 4H\left(\frac{n}{2}\right) + n$
$F(n) = 3F\left(\frac{n}{3}\right) + n^2$
$I(n) = 4I\left(\frac{n}{2}\right) + n^2$
$T(n) = aT\left(\frac{n}{b}\right) + f(n)$
$A(n) = \sqrt{n}\lg n$
$B(n) = n$
$C(n) = n^2$
$D(n) = \sqrt{n}$
$E(n) = n\lg n$
$F(n) = n^2$
$G(n) = n^2$
$H(n) = n^2$
$I(n) = n^2\lg n$

$A(n) = 2A\left(\frac{n}{4}\right) + \sqrt{n}$		
$D(n) = 3D\left(\frac{n}{3}\right) + \sqrt{n}$	$E(n) = 3E\left(\frac{n}{3}\right) + n$	$F(n$
$G(n) = 4G\left(\frac{n}{2}\right) + \sqrt{n}$	$H(n) = 4H\left(\frac{n}{2}\right) +$	$I(n$

(A) $2A\left(\frac{n}{4}\right) + \sqrt{n}$



work:

$\frac{n}{4^h} = 1 =$

$A(n) = \sqrt{n}\lg n$	$B(n) = n$
$D(n) = n$	$E(n) = n\lg n$
$G(n) = n^2$	$H(n) = n^2$