Sub-additive Valuation functions

Partition $\leq_p$ Subs $= \{ \langle M, n, (v_1, v_2, \cdots, v_n), K \rangle$

There exists an allocation $(A_1, A_2 \cdots A_n)$ of the items M

s.t. $\sum_{i=1}^{n} v_i(A_i) \geq K$

$\langle x_1, \cdots x_n \rangle \mapsto \langle M, 2, (v_1, v_2), K \rangle \rightsquigarrow \sum_{i=1}^{n} x_i$

(polynomial) $x_1, \cdots x_n$

$v_i(A_i) = \left(\sum_{x_j \in A_i} x_j, \frac{\sum_{i=1}^{n} x_i}{2} \right)$

Partition = NP-hard $\Rightarrow$ Subs : NP-hard.

$A = \{$ give everything to player $i \mid 1 \leq i \leq n \} \cup$

{ give every player just one item }

Colby

