CS 386 – Lecture 12
Introduction to Elections

October 28, 2026

1 SAT / 3SAT History Lessons

1.1 Historically

Brute force $\sim O^*(2^n)$

'85	1.618^n
'97	1.579^n
'99	1.505^n
'00	1.481^n
'04	1.473^n
'11	$\left(\frac{4}{3}+\varepsilon\right)^n$

$$
\begin{aligned}
b^n &= \left(2^{\log_2 b}\right)^n \\
&= 2^{\log_2 b \cdot n} \qquad \text{[Exponential Time Hypothesis]} \\
&= 2^{\Omega(n)}
\end{aligned}
$$

$$\Downarrow$$

$$\text{P} \neq \text{NP}$$

1.2 Anthropic + OpenAI recently

$2^{O\left(\frac{n \log(\log n)}{\sqrt{\log n}}\right)}$ $\rightarrow$ actually an improvement.

Recently, OpenAI found $n(\log n)^{1-\frac{1}{2^{182}}}$ vs $n \log n$ for integer multiplication.
$\rightarrow$ improvement but not observable.

1.3 Complexity Zoo

P NP #NP

$\hookrightarrow$ NP to count # of solutions

Not NP:

$$\{\langle M, w\rangle \mid M \text{ halts on } w\}$$

$$\left\{\langle M, w, \underbrace{1111}_{t \text{ times}}\rangle \;\middle|\; M \text{ halts on } w \text{ in } 2^t \text{ steps}\right\}$$

2 Elections

2.1 Examples

- 2000 US presidential election in Florida (Most-preferred voting)
  $\rightarrow$ Is this election system good? Did the right person win?
- 2026 Maine gubernatorial election (Ranked choice voting)

2.2 Setup

$v_i \sim \pi(A)$: A permutation of the set of candidates.

$f : V \to A$: A social choice function takes in everyone's rankings of the candidates and spits out the winner.

$f : V \to \Pi(A)$: A social welfare function takes in everyone's rankings of the candidates and spits out a ranking of the candidates.

2.3 Properties of Voting Rules

1. **Unanimity:** If everyone's rankings are the same, just output that ranking.
2. **Pareto:** If everyone agrees $a > b$, then the output should have $a > b$.
3. **Monotonicity:** If $f(v_1, v_2, \ldots, v_i, \ldots, v_n) = a \neq a' = f(v_1, v_2, \ldots, v_i \to v_i', v_{i+1}, \ldots, v_n)$, then $a >_i a'$ and $a' >_{i'} a$.

4. **Independence of Irrelevant Alternatives (IIA):** The outcome between a & b should have no dependence on any other candidate.

   Example: If $\frac{2}{3}$ of voters prefer a to b, $\frac{2}{3}$ of voters prefer b to c then ...

v_1 :	v_2 :	v_3 :
A	C	B
B	A	C
C	B	A

2.4 Election Rules

1. **Plurality:** Whoever has most first-place votes, make them win. Recurse on remaining.
2. **IRV:** Set whoever has minimum 1st place votes last, recurse on remaining.
3. **Borda:** Give each candidate $(n - i)$ points for each i^{th} place vote.

2.5 Theorems

1. **Arrow's Theorem:** Every social welfare function when $|A| \geq 3$ that satisfies unanimity and IIA is a dictatorship.
2. Monotonicity $\Leftrightarrow$ IC.
3. **Gibbard–Satterthwaite Thm:** If f is a monotone/IC social choice function, when $|A| \geq 3$, then f is a dictatorship.

2.6 Additional Example

2 :	3 :	1 :	3 :
A	B	B	C
C	A	C	A
B	C	A	B

1. **Plurality:** $B > A > C$.
2. **IRV**: $A < B < C$.
3. **Borda:**

$$A : 10$$
$$C : 9$$
$$B : 8$$