

Nash Equilibrium: A specification of strategies for each player such that no player is incentivized to deviate from their strategy unilaterally.

Nash (50, 51): Every finite player where each player have a finite set of strategy must have a Nash Equilibrium, (could be randomized strategy)

Naming Number Game:

Rule: Player 1 and 2 both name a number. If sum is even player 1 wins, else player 2 wins.

		Player 2	
		E	O
Player 1	E	-1, 1	1, -1
	O	1, -1	-1, 1

No choice that is always dominant.

Player 1 is best off always going 50/50 on even or odd.

If both player choose even or odd at 50/50, then it is a Nash equilibrium.

Say Player 1 deviates and goes even 75% of time and odd 25%.

Say Player 2 goes p % even, then

$$E(\text{value of Player 2}) = \frac{3}{4}(1-p) - \frac{3}{4}p + \frac{1}{4}p - \frac{1}{4}(1-p) = \frac{1}{2} - p$$

Game of Chicken

		Player 2	
		T	W
Player 1	T	-100, -100	-5, 5
	W	5, -5	0, 0

Say Player 1 go T with p and W with $(1-p)$,

then $E(\text{Player 2 if T}) = -100p + 5(1-p)$ and $E(\text{Player 2 if W}) = -5p$

setting $E(p2 \text{ if T}) = E(p2 \text{ if W})$, then we find $p = \frac{1}{20}$

If $p < \frac{1}{20}$, Player 2 should go straight.

If $p = \frac{1}{20}$, Player 2 should turn.

If $p > \frac{1}{20}$, Player 2 should go straight at chance $\frac{1}{20}$.

Random Game

		Player 2		
		A q	B $(1-q)$	C
Player 1	(a) p	0, 5	5, 0	6, 1
	(b) $(1-p)$	3, 5	0, 3	1, 0

← Pure Nash Equilibria

$$E(p2 \text{ if A}) = 5(1-p), \quad E(p2 \text{ if B}) = 5p + 2(1-p), \quad E(p3 \text{ if C}) = 6p$$

If $E(p^2 \text{ if } A) = E(p^2 \text{ if } B)$, then $p = 3/8$

$E(p^1 \text{ if } (a)) = 3q$ and $E(p^2 \text{ if } (b)) = 2(1-q)$

then $q = \frac{2}{5}$ if $E(p^1 \text{ if } (a)) = E(p^2 \text{ if } (b))$

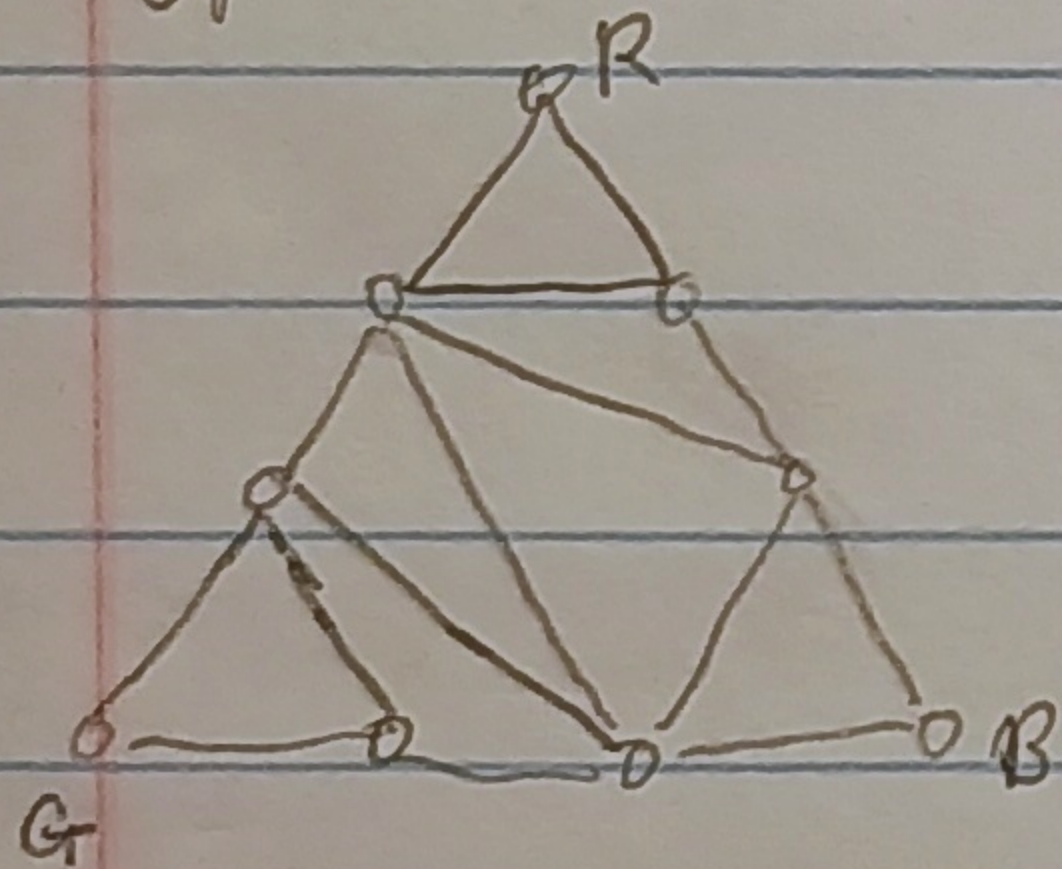
Big O for calculating all pure Nash is $n \cdot m(n+m)$

Big O for calculating all combination Nash is $2^n \cdot 2^m = 2^{n+m}$

Brouwer Fixed Point Problem

Let $f: [0,1]^n \rightarrow [0,1]^n$ where f is surjective, If f is continuous, then there must be a point x where $f(x) = x$. (Now, try to find this point x).

Sperner's Lemma



If you have a triangulated triangle where each vertex have assigned color RGB. If corner vertices are each a different color and each main edge have only two color, then there is always a smaller triangle with three different color vertices. (Try find this triangle).

End of the Line

Input: a directed graph where each vertex have at most one predecessor and one successor, along with one source vertex.

Find sink in $(\log n)^2$

PPAD: Polynomial Parity Arguments on Directed Graph

Nash Equilibrium computation is in PPAD.

The problems: Braker, Sperner, and End of the Line are all problems where we know such an object (whether that is a point, a triangle, or a vertex) must exist, but finding it is computationally hard. However, finding the Nash Equilibrium is even harder computationally. In fact, theoretically speaking, if a polynomial time solution to Nash Equilibrium searching was to be found, then through reduction, algorithms for the three problems above can also be solved.

