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9/24 Scribe Lecture Notes CS386

Goal: No finite alg. for envy free continuous cake-cutting
- Maybe Later: Selfridge-Conway finite bounded
Envy free for $n=3$

Given list of n nums., we can't find a target in $O(n)$. Proof will show similar conclusion.

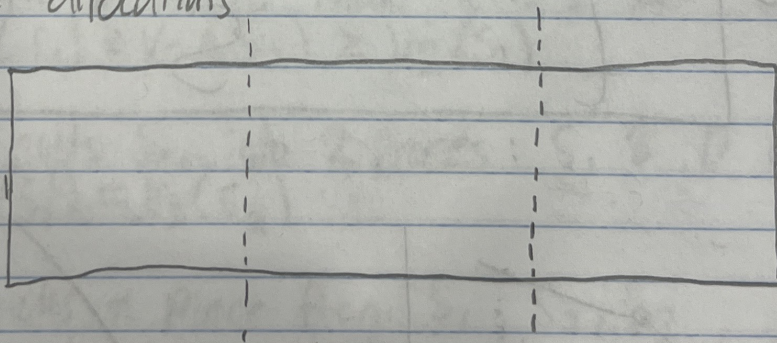
Protocols we play against:

1) Ask players to mark starting from the left side $x\%$ of the cake.

2) Make a cut at a mark that has been made.

★ Need to assume players are honest ★

We will construct a valuation function for each player that forces exactly 1 way to cut the cake into envy free allocations



Rigid Measure System:

Parameters x, y, s, t s.t. $s + 2t = 1$

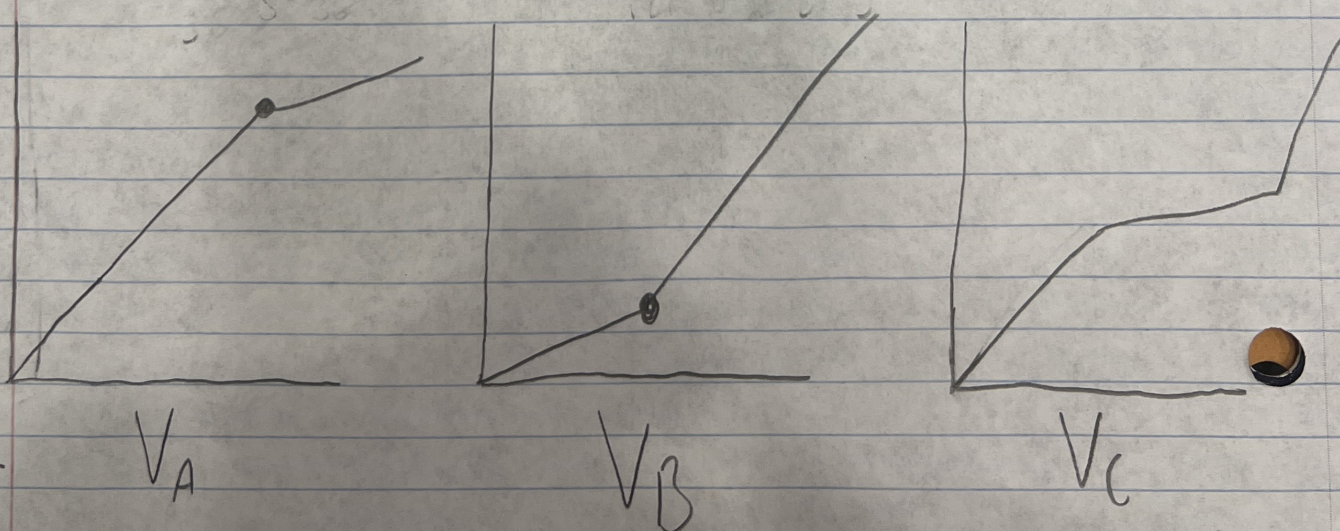
and valuation functions v_i $0 < s < \frac{1}{3} < t < \frac{1}{2}$

1) $\frac{\sqrt{2}}{2}l < v_i(s) < \sqrt{2}l$ $l = \text{length of } s$

2) The players' valuations are specific on the following intervals

	$[0, x]$	$[x, y]$	$[y, 1]$
V_A	t	t	s
V_B	s	t	t
V_C	t	s	t

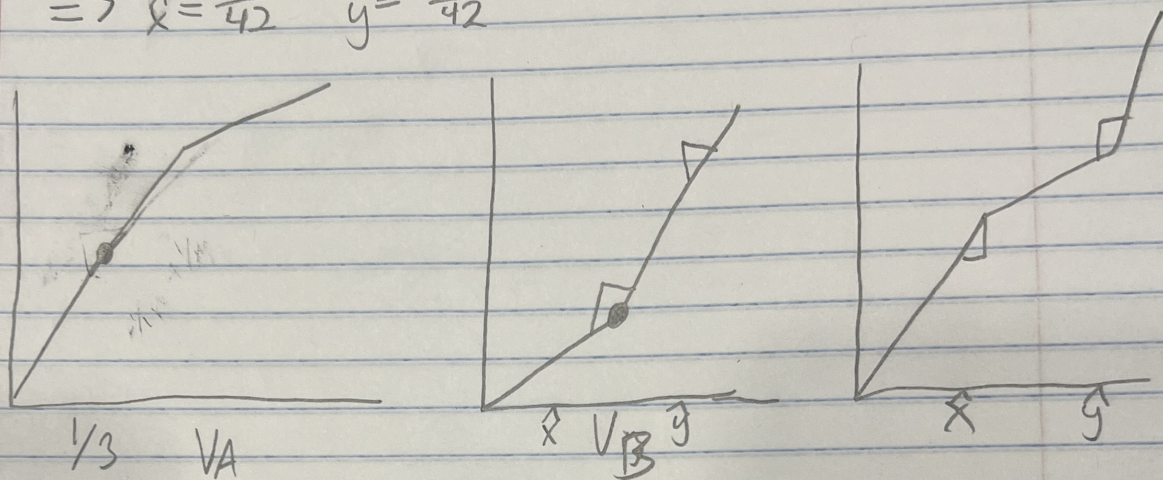
$x = \frac{1}{3}$
 $y = \frac{2}{3}$
 $s = \frac{4}{18}$
 $t = \frac{7}{18}$



η = the closest they had been so $= \frac{1}{18}$ for to x or y

$$\hat{t} = \frac{13}{36} \quad \hat{s} = \frac{10}{36} \quad V_A(\hat{x}) = \hat{t} \quad V_A(\hat{y}) = 2\hat{t}$$

$$\Rightarrow \hat{x} = \frac{13}{42} \quad \hat{y} = \frac{26}{42}$$



1. Player 1 cuts the cake into 3 pieces
2. Player 2 sorts those pieces so that $V_2(S_1) \geq V_2(S_2) \geq V_2(S_3)$
3. P_3 cuts S_1 into 2 pieces: S_1' & R so that $V_2(S_1') = V_2(S_2)$
4. P_3 picks a piece from S_1', S_2, S_3
 P_2 picks a piece but must pick S_1' if still available
 P_1 picks the last.
5. Let's call the person who selected S_1' P_A . call the other player who isn't P_1, P_B .
6. P_3 cuts R into 3 pieces.

7. P_A, P_i, P_B pick their choices in that order.