

Analysis of Algorithms
CS 375, Spring 2021

Problem Set: Dynamic Programming

Due **BY THE BEGINNING OF CLASS** Wednesday, May 5

- **READING:** From your textbook, please read Ch. 15, pages 359–360, 378–396; Ch. 24, pages 643–top of page 647; Chapter 25.2.
- To submit this Problem Set, the standard file naming and HW submission conventions for CS375 apply: Please put your answers in a PDF file named

`CS375_PS_DynamicProgramming-<userid>.pdf`

where `<userid>` is replaced by your full Colby userid (including your class year)—for example, my file would be called `CS375_PS_DynamicProgramming_eaaron.pdf`—and submit it to your `SubmittedWork` folder.

- **IMPORTANT:** Some of these exercises may build upon topics covered in class meetings after the day this is assigned; they are included here “early” so you can see all of the exercises on this assignment.
- Please recall the guidelines for algorithms given on previous HW assignment sheets. They continue to apply to all HWs in CS375.
- In general, there may be multiple correct ways of presenting an algorithm, although excessively inefficient or inelegant solutions may not receive full credit. If you have questions about whether your proposed solution is excessively inefficient or inelegant, please ask your Prof.!
- *A general note for CS375:* Please present answers cleanly and **explain them clearly and thoroughly**, giving all details needed to make your answers easy to understand; typed-up (rather than handwritten) answers are especially appreciated. (Feel free to talk with your Prof. or TA’s about using L^AT_EX to typeset your answers!) Graders may not award full credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

In general in CS375, unless explicitly specified otherwise, answers should be accompanied by explanations. Answers without explanations may not receive full credit. Please feel free to ask me any questions about explanations that might come up!

Exercises

1. A pair of rabbits, one male and one female, is placed in a large sealed enclosure. How many pairs of rabbits will be there in a year if the initial pair are newborn and all rabbit pairs are not fertile during their first month of life but thereafter give birth to one new male / female pair at the end of every month?

Please explain your answer; a short explanation (3–4 sentences or so) could suffice, as long as it contains the ideas underlying the solution.

2. In chess, a rook can move horizontally or vertically to any square in the same row or in the same column of a chessboard. Find the *number of shortest paths* by which a rook can move *from the bottom-left corner of a chessboard to the top-right corner*. (The length of a path is measured by the number of squares it passes through, including the first and the last squares.)

Solve the problem by a dynamic programming method. That is, come up with a relevant recurrence—i.e., a recursive definition of the relevant values for a solution, as we’ve seen in the examples from class—and using dynamic programming techniques, calculate the solution. Please show your work by giving an 8×8 table (each element in the table represents the corresponding square on the chessboard), where each entry in the table should be the number of paths from the bottom-left corner to that square on the chessboard; therefore, the answer to this exercise will be the number in the top-right element of your table.

Note that you do not need to provide a shortest path from one corner to the other, just the number of shortest such paths. Please be sure to include an explanation of the correctness of your recurrence; you do not need to explain how you used the generated table from the recurrence.

3. CLRS Exercise 15.4-1 (pg. 396). This exercise is intended to give you practice with the algorithm in Section 15.4—for your answer, please show the table constructed (see Figure 15.8) for this example, and give the LCS derived from that table (following the methods in the textbook). Although in general, explanations should accompany answers in CS375, for this exercise, no other explanation is needed, just show the table and give the LCS derived from it.