

Analysis of Algorithms
CS 375, Spring 2021

Problem Set: Recurrences

Due **BY THE BEGINNING OF CLASS** Monday, April 12

- From your textbook (CLRS), please read all of Chapters 2 and 3, and pages 65–67 and 88–97 from Chapter 4. Although not all of it will show up on CS375 exercises, it is all good material to know.
- To submit this Problem Set, the standard file naming and HW submission conventions for CS375 apply: Please put your answers in a PDF file named

CS375_PS_Recurrences_<userid>.pdf

where <userid> is replaced by your full Colby userid (including your class year)—for example, my file would be called `CS375_PS_Recurrences_eaaron.pdf`—and submit it to your `SubmittedWork` folder.

- Unless otherwise specified, exercises will be from the CLRS textbook and will be named on HW assignments by exercise number used in the book.
- **IMPORTANT:** Some of these exercises may build upon topics covered in class meetings after the day this is assigned; they are included here “early” so you can see all of the exercises on this assignment.
- Please recall the guidelines for algorithms given on previous HW assignment sheets. They continue to apply to all HWs in CS375.
- In general, there may be multiple correct ways of presenting an algorithm, although excessively inefficient or inelegant solutions may not receive full credit. If you have questions about whether your proposed solution is excessively inefficient or inelegant, please ask your Prof.!
- *A general note for CS375:* As always, please present answers cleanly and **explain them clearly and thoroughly**, giving all details needed to make your answers easy to understand; typed-up (rather than handwritten) answers are especially appreciated. (Feel free to talk with your Prof. or TA’s about using L^AT_EX to typeset your answers!) Graders may not award full credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

In general in CS375, unless explicitly specified otherwise, answers should be accompanied by explanations. Answers without explanations may not receive full credit. Please feel free to ask me any questions about explanations that might come up!

Exercises

1. Using the *unwinding* method from class, solve the following recurrence relations and give the Θ class of the solution.
 - (a) $T(n) = 3T(n - 1)$ for $n > 1$; $T(1) = 4$.
 - (b) $T(n) = T(n - 1) + n$ for $n > 0$; $T(0) = 0$.

For each of the above, show your work in doing the “unwinding.” To show a full understanding of the unwinding method, please be sure to show a k ’th step—a step at some representative k ’th step in the unwinding, in terms of k rather than some specific number, as demonstrated in the lecture of March 31 and shown in lecture notes—which shows the pattern upon which your solution is based.

2. Solve the following recurrence **showing your work, using the recursion tree method** and give the Θ class of the solution.

$$T(n) = 3T(n - 1) \text{ for } n > 1; T(1) = 4.$$

Please show your work by including a table of the following form. The table should include one row for each level in the tree; be sure to show a representative k ’th level to illustrate the relevant pattern, similar to the k ’th step in the unwinding method.

level	# nodes	work per node	total work
0	nodes on level 0	work per node on level 0	total work on level 0
1	nodes on level 1	work per node on level 1	total work on level 1
2	nodes on level 2	work per node on level 2	total work on level 2
...	...	...	...
k	nodes on level k	work per node on level k	total work on level k
...	...	...	...
$\langle \text{last level} \rangle$	nodes on $\langle \text{last level} \rangle$	work per node on $\langle \text{last level} \rangle$	total work on $\langle \text{last level} \rangle$

Then, explain how you use information from your table to arrive at a final answer. Recall that you can use high-level explanations for Θ analysis; you don’t need to give witnesses for the values of the relevant constants.

NOTES: You might want to explicitly include the second-to-last level in your table, as well as the last level, because the last level is sometimes so different from the remainder of the table that it does not fit into the pattern. Also, you are also welcome to include a picture of the recursion tree as part of your answer if you’d like, but it is not required for the exercise.

3. Give Θ bounds for the following recurrences. *Be sure to use the Master Theorem for these exercises.* As always, be sure to give a brief explanation of your answers—here, that will include the values for each relevant variable in the Master Theorem, what case of the Theorem you are applying, and a brief explanation of how you know what case to apply.

(a) $T(n) = 4T(n/2) + n^2, T(1) = 1.$

(b) $T(n) = 4T(n/2) + n^3, T(1) = 1.$

4. Give Θ bounds for the following recurrence, using any of the three methods introduced in class (unwinding, recursion tree, Master method).

$$T(n) = 3T(n/2) + n \lg n, T(1) = 1$$

As always, be sure to give a brief explanation (with appropriate details) of your answer.