

CS486 Assignment 2

Due February 21st 10PM ET

1. This question is about the Minimal Cost Online Match-Making Problem with Delays, as defined in class:

Problem 1 (Minimal Cost Online Match-Making Problem with Delays).

Input: An online sequence of players $p_1, \dots, p_{2m}$ where

- each player is of team either R or B
- there are a total of m players each on team R and team B (this value m is unknown and is never explicitly revealed)
- matches can only be created between players of opposite teams.
- upon arrival of a new player v , the relative “skill difference” $d(u, v)$ u and all previously arrived players v each player is of team either R or B are revealed; these differences satisfy the triangle inequality

Mechanism: The mechanism is allowed at any time to match currently unmatched players of opposite teams.

Goal: The objective function is to minimize the cost of every match $\{u, v\}$, defined to be the sum of both players’ waiting times and their skill difference.

- i) I forgot to mention it, but the ‘natural’ GREEDY algorithm is really bad for this: specifically, the algorithm that creates the “best” match currently available, without ever delaying. Show that this algorithm is $\Omega(2^m)$ competitive. **Hint:** Think back to last semester... This problem is extremely similar to one from near the end of last semester...

The algorithm we analyzed was called GREEDYDUAL. To define it, we used the following value

$$\text{OPTCOST}(\{u, v\}) = d(u, v) - (\text{the difference in arrival times of } u \text{ and } v),$$

denoting the cost an optimal solution would pay for creating a particular match between players u and v . The algorithm also maintains a function $\mathcal{A} : V_t \rightarrow \mathcal{P}(V_t)$ where V_t denotes the set of players who have arrived by time t that maps players to their “active set”. The definition of the algorithm is as follows:

GREEDYDUAL

- Upon arrival of a player v at time t :
 - i) Create a variable $y_S = 0$ for every $S \subseteq V$ where $v \in S$
 - ii) Set $\mathcal{A}(v) = \{v\}$
- When $\sum_{S: e \in \delta(S)} y_S = \text{OPTCOST}(e)$ for any edge $e = \{u, v\}$ (where $\delta(S)$ denotes the edges that have an endpoint in S and an endpoint outside of S)
 - i) Create $S = \mathcal{A}(u) \cup \mathcal{A}(v)$
 - ii) For every $w \in S$: set $\mathcal{A}(w) = S$
 - iii) Create matches of any possible unpaired players in S
 - iv) (Unnecessary but useful for analysis) ‘mark’ the edge e
- Always:
 - (a) For every set $S \in \mathcal{A}[V_t]$ that contains an unmatched player (where t is the current time): increase y_S at the same rate as time passes.

- ii) Show that GREEDYDUAL is $O(1)$ competitive on the input you used in i above.
- iii) I was going to make you do the stuff I mentioned in class, but I decided it was more important that you figure out what you’re doing this semester, so I’ve commented it out. If you want to flex, you can go uncomment the questions in the attached tex file and answer them anyway.

2. What are you planning on working on this semester? I’m expecting an explicit problem statement (like I’m phrasing problems in these assignments, so describe a specific input/mechanism/objective) **OR** a specific class of problems (I think this only applies to MG&EB at the moment?)